Performance Based Learning and Assessment Task

Koi Pond

I. ASSESSMENT TASK OVERVIEW & PURPOSE:
The students will design a koi pond to certain specifications for a client of their landscaping firm, applying their understanding of perimeter, area and volume.

II. UNIT AUTHOR:
Savannah Montgomery, Magna Vista High School, Henry County Schools
Linda Woodford, Franklin County High School, Franklin County Schools

III. COURSE:
Geometry

IV. CONTENT STRAND:
Geometry: Problem Solving with 2- and 3-Dimensional Figures

V. OBJECTIVES:
The student will be able to:

- Combine a minimum of 3 shapes (polygons and portions of circles) to make an outline of a figure
- Calculate the perimeter of a figure comprised of at least 3 different shapes
- Calculate the area of a figure comprised of at least 3 different shapes
- Calculate the volume of three dimensional figures
- Draw a 2 dimensional representation of a figure comprised of at least 3 different shapes
- Make a 3 dimensional model of a figure comprised of at least 3 prisms and/or portions of cylinders.

VI. REFERENCE/RESOURCE MATERIALS:
Graph paper for drawing, paper to perform calculations, poster board for 3 dimensional model, ruler, scissors, tape, calculator, access to PowerPoint. Optional resources: computer programs such as Geogebra or Paint.

VII. PRIMARY ASSESSMENT STRATEGIES:
Students will complete a self-assessment checklist based on a provided rubric. The teacher will use the same rubric to assess student performance based on correct mathematical computations, correct graphical and 3D representations, adherence to prescribed parameters, and a neat presentation of results.

VIII. EVALUATION CRITERIA:
Assessment lists, corresponding rubric, and a sample benchmark are included.

IX. INSTRUCTIONAL TIME:
This activity will take four ninety-minute class periods.

Koi Pond

Strand
Geometry: Three Dimensional Figures

Mathematical Objective(s)
The overall mathematical goal of this activity is for students to design a koi pond using at least three different shapes for the outline, and a minimum of two different depths. They will have to calculate the perimeter of their pond, the area of the pond surface, and the volume of the pond in cubit feet and gallons. Their product will be a drawing of their pond, a 3 dimensional representation of the pond, and a short PowerPoint presentation.

Related SOL

- 8.7 The student will
  a) investigate and solve practical problems involving volume and surface area of prisms, cylinders, cones, and pyramids; and
  b) describe how changing one measured attribute of a figure affects the volume and surface area.
- 8.9 The student will construct a three-dimensional model, given the top or bottom, side, and front views.
- 8.11 The student will solve practical area and perimeter problems involving composite plane figures.
- G.13 The student will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.

NCTM Standards

- Precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties;
- Analyze properties and determine attributes of two- and three-dimensional objects;
- Draw geometric objects with specified properties, such as side lengths or angle measures;
- Use two-dimensional representations of three-dimensional objects to visualize and solve problems such as those involving surface area and volume;
- Recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life.
- Draw and construct representations of two- and three-dimensional geometric objects using a variety of tools;
- Use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture.
- Apply and adapt a variety of appropriate strategies to solve problems
- Communicate mathematical thinking coherently and clearly to peers, teachers, and others

Materials/Resources

Students will need the following materials to complete the activity:

- Koi Pond Design Guidelines
- Paper to perform calculations
- Graph paper
- Poster board
- Ruler
- Scissors
- Tape
- Computer Programs such as Geogebra or Paint (optional)
- Calculator
- Access to PowerPoint

Assumption of Prior Knowledge

Students should have basic knowledge of plane figures and be able to combine them into a shape on graph paper. They should know how to calculate perimeter and area of plane figures and volume of prisms and cylinders. They should be able to create a 3 dimensional model.

As students design their koi ponds, they should discuss what will make a unique and pleasing geometric design. They will talk about polygons and semicircles and how to create a pond that will hold at least 1000 gallons of water. They should also be considering the feasibility of their final design as an actual koi pond, and should be discussing whether it would be a realistic size and shape to fit into a person's yard.

Students may find it difficult to visualize the actual 3 dimensional pond, as compared to the surface shape, and may need assistance to determine the size and shape of the pieces that will make up their 3 dimensional model. Any groups needing help may be given suggestions.

Introduction: Setting Up the Mathematical Task

- The teacher will introduce the task by asking students "How many of you know what koi are? Have you ever seen a koi pond?" The teacher could show pictures of koi and tell the size of the koi (when full grown) that will swim in the pond. More information about koi and their needs may be found on this website: http://www.peteducation.com/article.cfm?c=16+1917&aid=2903. In this activity, students will apply their knowledge of perimeter, area, and volume in order to design a koi pond. Students should be comfortable with plane figures, using formulas, and making a 3 dimensional model from cardboard or poster board.

- This lesson should take four class periods. During the first day, the teacher will prompt students to recall their knowledge of area and volume formulas. For the task of designing the koi pond, students need to be able to put several plane figures together to make the outline of the pond they want to create. Some prompts could include having them start with one section of the pond, and then add other sections to it. Teachers could have students search for pictures of koi ponds on the internet to get ideas. The teacher may have to remind students that the actual pond is 3 dimensional, and they must plan the depth of each section in addition to choosing the outline of the pond. This task should be completed in groups of no more than three to encourage students to discuss ideas and make decisions as a group. After completing the design, students will calculate the perimeter of the pond, area of the surface of the pond, and volume of the pond in both square feet and gallons.

- On the second day, students should complete their three dimensional models and plan their class presentation. Students should include in their presentation the two dimensional drawing of the outline of the pond, the final perimeter, area and volume, and a display of their three dimensional model.

- On the third day, students should finish their three dimensional models and make their PowerPoints.

- On the fourth day, groups should make their presentations to the client (the class).

- Questions to prompt students: What would be a pleasing shape that your client will like? Can the fish get to all sections of the pond easily? Can your pond accommodate adult koi? Will the pond be a reasonable size to put in someone's yard?

Student Exploration

- Students are presented with the project and begin working on their design and calculations in their groups.
- Students build their scale model.
- Students create a PowerPoint and plan their presentation
- Students present their projects to the class

Student/Teacher Actions:

- As groups design their koi pond, students should be identifying plane figures that will make an interesting geometric shape and designing a pond that meets the specified criteria. Students may need help in building their three dimensional models.
- The teacher should ensure that the students are using the provided guidelines, and doing correct calculations. The teacher can ask the students to justify their choices if the teacher identifies issues with the pond outline in order to help the students correct their own mistakes.
- Students may forget to include the depth of each section and focus only on the two dimensional surface of the pond. Some questions to address those errors: Is the pond flat like paper? Does it need depth? What formulas should we use in that case? How can we find the area or volume of such an odd shape?
- Have the students identify features that are good and features that are not good for a koi pond. Consider things like space for the fish to swim even when they are full grown, the ability of fish to reach all sections of the pond, and a depth that is comfortable for the fish but also allows for the client to enjoy watching the fish.
- This task can be done with or without the use of technology (on graph paper, or with calculators or computer graphing software). If time is an issue, the PowerPoint presentation could be dropped and students just do a show-and-tell.
- An extension of this project might be to combine students with students in a science or horticulture class and give them the pond designs and ask the students to come up with pond plants that koi would like and that can be grown at each depth. In addition, they could choose plants to place around the border of the pond to provide shade for the koi while also allowing access to the pond for the client to view the fish.

Monitoring Student Responses

Students should present a pond design that meets the objectives both mathematically and physically for a realistic koi pond. Students should discuss their designs with each other to facilitate a conversation about what would make a shape work for the requirements and the objectives. Students who are having difficulties should be prompted to start with one shape and try to add a second shape and third shape. The teacher will walk around and facilitate as needed. Students will be provided with a rubric to assist them in meeting the task objectives. Students will summarize their activity by presenting the pond design and calculations to the class.

For closure, have a class discussion about what kind of math was used in this real life situation and ask students what was their favorite part of the project.

Assessment List and Benchmarks

Students will complete a design for a koi pond based on the given guidelines. Students will self-assess their work using the rubric provided. The teacher will use the same rubric to assess the students' performance. Students will also present their koi pond to the class. Students could provide feedback to each group by identifying one good aspect of the design and one suggestion to make it better.

Koi Pond: Design Guidelines

You are a part of a group of landscapers. You have a client that wants to build a koi pond in his back yard. The client wants a unique geometric design so you must use at least three unique shapes to incorporate into the design (remember these are three dimensional figures because you are creating a pond). If you use a cylinder or a semi-cylinder, use 3.14 for pi. At least one section of the koi pond needs to be deeper than the others, and koi prefer their ponds to be at least 1,000 gallons (1 ft^3 = 7.48 gallons). The minimum depth is 3 ft. You must do a drawing of your pond, with dimensions (feet) and calculations included. You will need to calculate how much space in the backyard you will use, how much border you will need for the pond, how deep you will need to dig to accommodate the pond, and how many gallons of water the client will need to fill the pond. You will also have to build a scale model of your pond using the materials provided in class. You are welcome to bring in your own materials as well. Use the scale 1in=1ft for your model. You will have to present your design to the client (class). In your presentation, you will need to show the scale model of the pond you designed and all the calculations specified above (in a short PowerPoint presentation).

Koi Pond: Grading Rubric

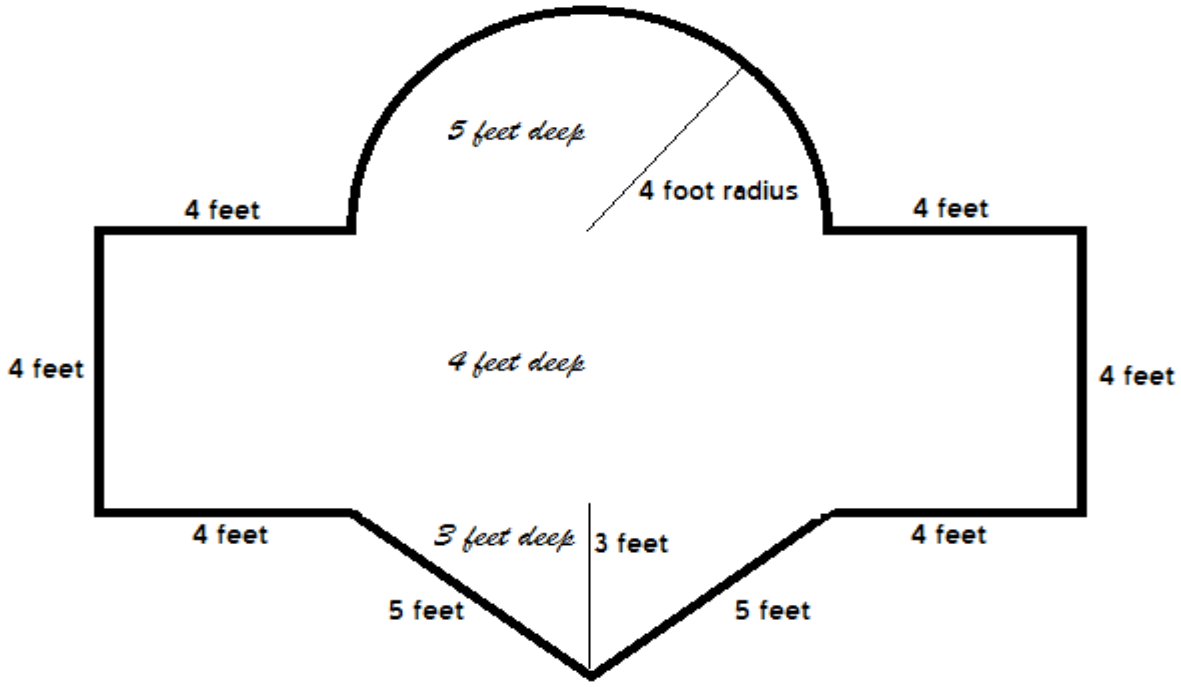
This project is worth 20 points, 12 points for your computation and 8 points for your presentation. Below is a rubric break down so you know EXACTLY what is expected. The teacher will grade it (teacher points) and the student will evaluate themselves (student points). Teacher comments and total points are at the bottom.

Computation-12 points				
Drawing of the design-2 points				
2 points: student does an accurate and neat drawing of the design	1 point: student does a somewhat accurate drawing of the design	0 points: student did not include a drawing	Teacher Points:	Student Points:
Dimensions-2 points				
2 points: student includes dimensions and correct units of their shapes clearly labeled	1 point: student includes some dimensions of the shapes	0 points: student did not include dimensions of the shapes	Teacher Points:	Student Points:
Perimeter- 2 points				
2 points: student shows step by step how to find the perimeter of the pond, including correct units	1 point: student is not detailed, makes a mistake finding the perimeter, may not include correct units	0 points: student did not include perimeter	Teacher Points:	Student Points:
Area- 2 points				
2 points: student shows step by step (including formulas) how to find the combined area of the pond, including correct units	1 point: student is not detailed, makes a mistake finding the area, may not include correct units	0 points: student did not include area	Teacher Points:	Student Points:
Volume- 2 points				
2 points: student shows step by step (including formulas) how to find the combined volume of the pond, including correct units	1 point: student is not detailed, makes a mistake finding the volume, may not include correct units	0 points: student did not include volume	Teacher Points:	Student Points:
Depth- 2 points				
2 points: student identifies the correct depth of the pond	1 point: student identifies the incorrect depth	0 points: student did not include depth	Teacher Points:	Student Points:

Presentation- 8 points					
Scale Model- 3 points					
3 points: student puts effort into building their scale model, has all the dimensions labeled with the correct units	2 points: student puts some effort into building their model, has all of the dimensions labeled	1 points: student builds a scale model, does not include the dimensions	0 points: student did not build a scale model	Teacher Points:	Student Points:
Presentation- 3 points					
3 points: student includes everything needed in a neatly designed PowerPoint slide and presents everything to the class	2 points: student includes everything in a PowerPoint slide and presents everything to the class	1 point: student does not include everything in the PowerPoint slide and presents it to the class	0 points: student does not prepare a presentation	Teacher Points:	Student Points:
Completed Project on Time- 2 points					
2 points: student turns in the project complete and on time	1 point: Student turns in the project late		0 points: student does not turn in a project	Teacher Points:	Student Points:
Comments:					

Total: /20

Koi Pond: Benchmark



Perimeter of Pond: Semi-Circle: $\frac{1}{2}(2\pi r)=\frac{1}{2}(2)(3.14)(4)$ =12.56 feet

Segments: $6\times 4+2\times 5=34$ feet

Total perimeter: 46.56 feet

Area of Pond: Rectangle: $bh=16\times 4=64$ square feet

Triangle: $\frac{1}{2}bh=\frac{1}{2}(8)(3)=12$ square feet

Semi-Circle: $\frac{1}{2}\pi r^2=\frac{1}{2}(3.14)(4)^2=25.12$ square feet

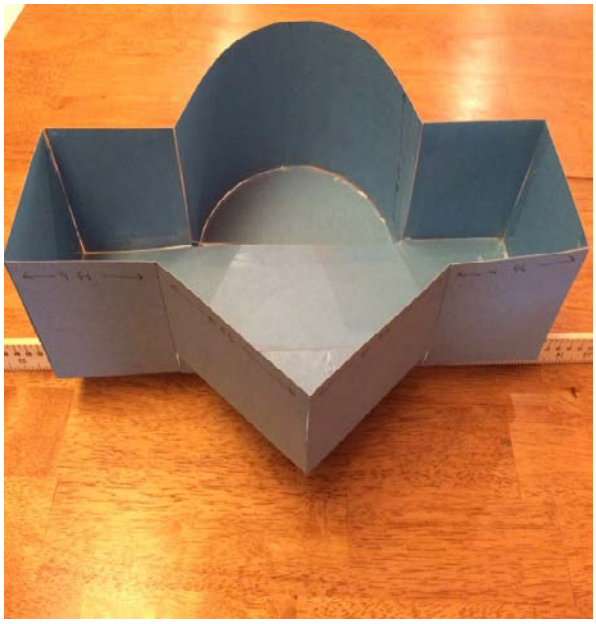
Total area: 101.12 square feet

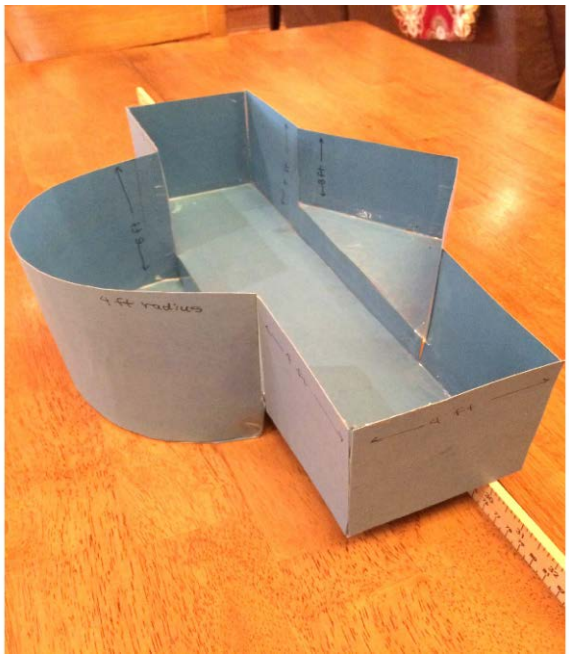
Volume of Pond: Rectangular Prism: $lwh=16\times 4\times 4=256$ cubic feet

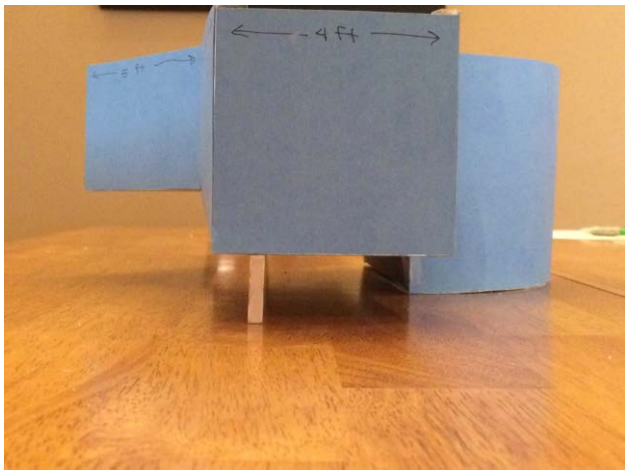
Triangular Prism: $Bh=\left(\frac{1}{2}\times 8\times 3\right)\times 3$ = 36 cubic feet

Semi-Cylinder: $\frac{1}{2}Bh=\frac{1}{2}(\pi r^2)h=\frac{1}{2}(3.14\times 4^2)\times 5$ = 125.6 cubic feet

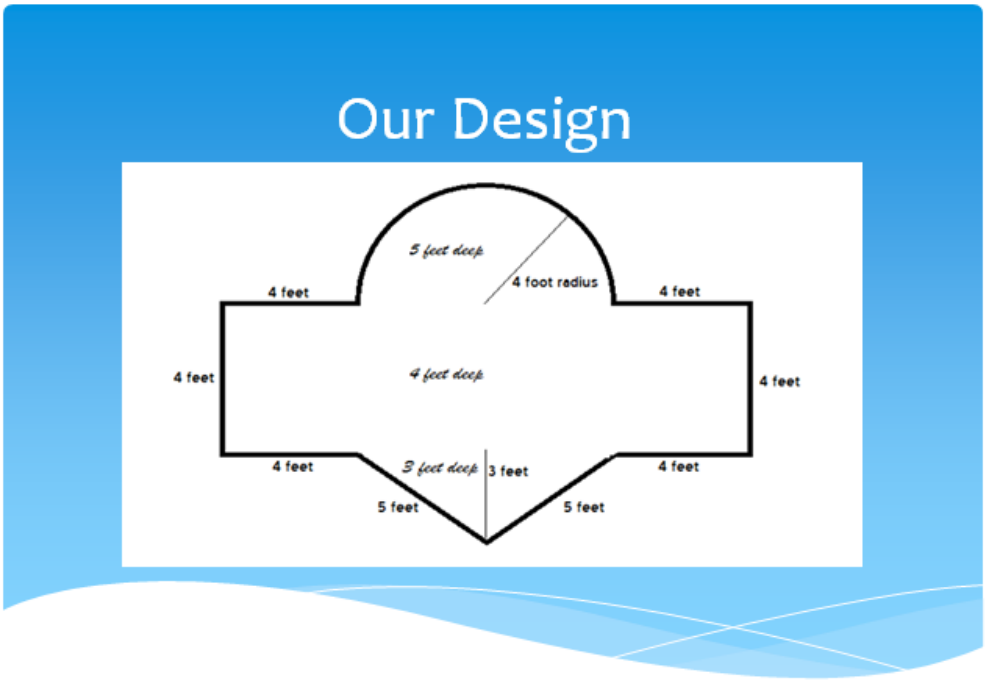
Total volume: 417.6 cubic feet $\times 7.48=3123.65$ gallons



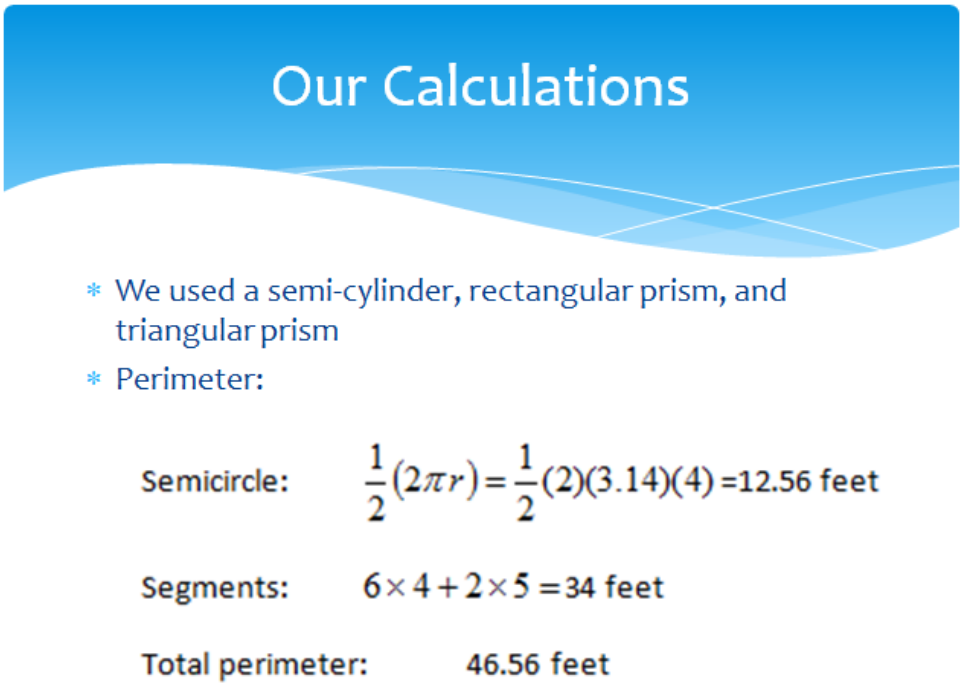

4 ft radius
4 ft


4 ft
5 ft

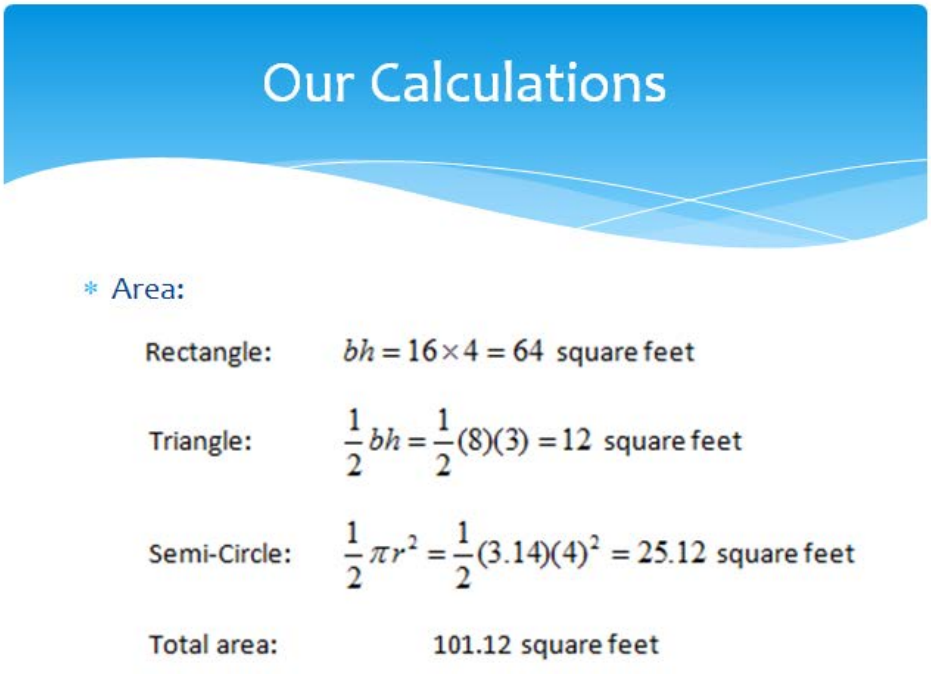
Student Power-Point Presentation:



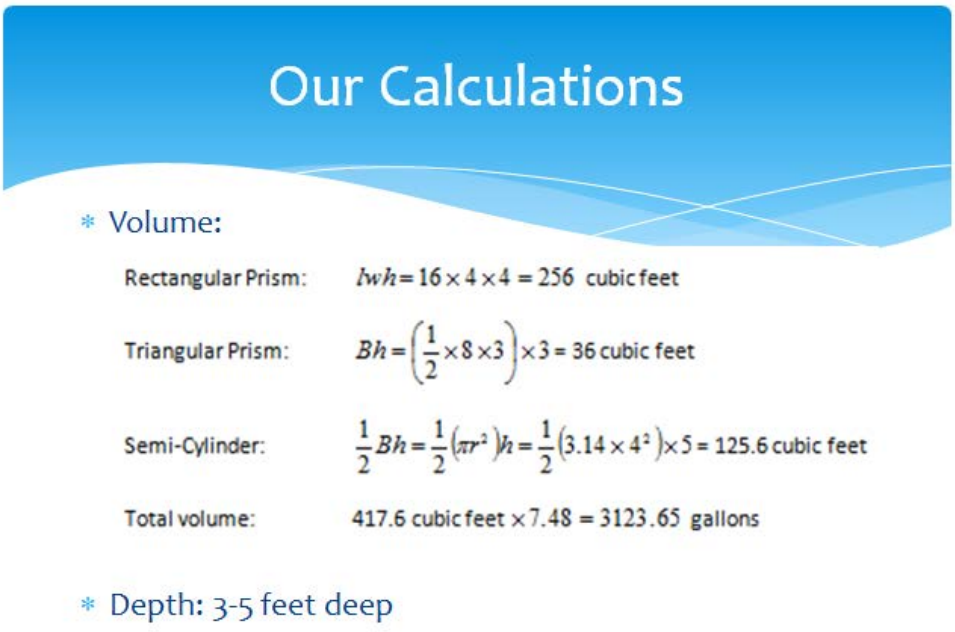
Slide 1: Hello customer. My partner and I are here to present to you our Koi Pond design.



Slide 2: We used a semi-cylinder, a triangular prism, and a rectangular prism.
For the perimeter of the pond we would need 46.56 feet of decoration for around the pond. We found this by adding up all the side lengths, and then for the semi-circle we took half of the circumference of the circle and added that in as well.



Slide 3: We will need to use 101.12 square feet of yard to build the pond. For this we found the area of the rectangle to be 64 feet, the area of the triangle to be 12 square feet, and the area of the semi-circle to be 25.12 square feet. We added it all up to get the total area that the top of the pond will cover in the yard.



Slide 4: You will need to supply 3,123.86 gallons of water for the pond. We found the volume of the rectangular prism to be256 cubic feet, the triangular prism to be 36 cubic feet, and the half cylinder to be125.6 cubic feet. Then we converted cubic feet to gallons. We will also need to dig a hole that ranges from 3 to 5 feet deep for the pond.

Slide 5: Can’t wait to do business with you!! Thank you!